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Short Communication

Dynamic characteristics of the out-of-plane vibration for an axially moving membrane

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Abstract

The dynamic characteristics of the out-of-plane vibration for an axially moving membrane are investigated in this study. The equations of in-plane and out-of-plane motions are derived by using the extended Hamilton principle. Two different types of boundary conditions are considered at the rollers: one is the condition of no friction and the other is the condition of no slipping. After discretizing the equations by the Galerkin method, the natural frequencies and mode shapes are computed. From the computation results, it is found that the translating speed, boundary conditions and aspect ratio of the membrane have effects on the natural frequencies, mode shapes and stability for the out-of-plane vibration of the moving membrane.

1. Introduction

Axially moving membranes are found in various engineering applications such as paper manufacturing, magnetic tape recording and so on. In these applications, the out-of-plane

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vibration of a membrane as well as the in-plane vibration is important in an axially moving membrane. In some circumstances, the out-of-plane vibration is more important than the in-plane vibration. For example, the most critical issue in paper industry is to reduce the out-of-plane vibration, because the severe out-of-plane vibration limits operation speeds resulting in low productivity of paper machines.

Besides moving membranes, various types of moving materials have been studied. The extensive research for axially moving materials can be found in a review article by Wickert and Mote [1]. Examples of the studies on axially moving beams, plates and strings are as follows. The vibration and stability of a band saw were studied with an axially moving plate model by Ulsoy and Mote [2]. Considering nonlinearity, the free vibration of an axially moving strip was examined by Thurman and Mote [3]. The vibration and the dynamic responses for a belt were presented by Kim and Lee [4]. Assuming the axial velocity to be periodic, the transverse vibration of an axially accelerating string was studied by Pakdemirli et al. [5]. The vibrations of an axially moving string with geometric nonlinearity have been studied by Chung et al. [6].

For axially moving membranes, Neimi and Pramila [7] analyzed the transverse vibrations of an axially moving membrane submerged in ideal fluid using the finite element method. They studied the effects of the element mesh density, the truncation distance and various lumping techniques on the accuracy of computation. Koivurova and Pramila [8] presented a theoretical and numerical formulation for a nonlinear axially moving membrane. They investigated geometrically nonlinear effects such as large displacements, variation of membrane tension and variation in axial velocity due to deformation. Recently, Shin et al. [9] studied the free in-plane vibration of an axially moving membrane considering the effects of both the translating speed and aspect ratio of the membrane. They discussed the effects of the translating speed, aspect ratio, and boundary conditions on the in-plane vibrations of the moving membrane.

In this paper, the out-of-plane vibration characteristics of the moving membranes are studied considering two kinds of in-plane boundary conditions introduced in Ref. [9]. Under the assumption that the in-plane motion is in a steady state and the out-of-plane motion is a dynamic state, the equations of in-plane and out-of-plane motions are derived from the extended Hamilton principle [10]. During the derivation, the geometric nonlinearity due to large displacements is included. After the derived equations are discretized by the Galerkin method, the natural frequencies and mode shapes are computed from the discretized equations. Based on numerical computations, the effects of the translating speed, boundary conditions and aspect ratio on dynamic characteristics are analyzed for the out-of-plane vibration of the moving membrane.

2. Equations of motion

Fig. 1 shows a schematic plot of an axially moving membrane with width b when the moving membrane has transverse vibration called the out-of-plane vibration. The membrane is supported by two pairs of rollers that are set apart from each other by length L. It is assumed in this paper that the membrane is translated in the x direction with an axially translating speed V and is subjected to a uniform tension per unit area T at the both ends.

The motion of the membrane can be determined by the displacements in the xyz coordinate system that is a space-fixed inertial frame. When point $P(x, y)$ moves to point $P'(x', y')$ due to

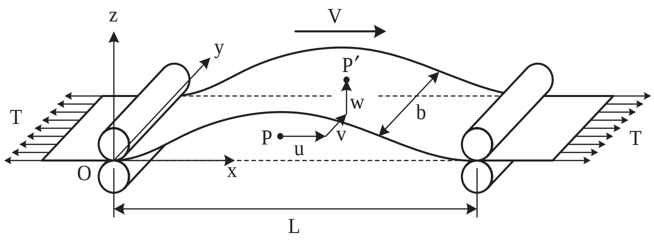

Fig. 1. Model of an axially moving membrane with the translating speed V.

deformation of the membrane, the motion of point P may be represented by the longitudinal displacement u, the lateral displacement v and the transverse displacement w. As shown in Fig. 1, the displacements u, v and w correspond to the displacements in the x, y and z directions, respectively.

A main assumption of this paper while deriving the equations of motion is that the out-of-plane motion is in a dynamic state while the in-plane motion is in a steady state. Note that the in-plane and out-of-plane motions are described by the in-plane displacements, u and v, and the out-of-displacement w, respectively. Since the in-plane stiffness is much higher than the out-of-plane stiffness, this assumption is acceptable and reasonable. Examples using this kind of assumption can be found in the papers regarding spinning disks [11–13]. Under the assumption mentioned above, the longitudinal and lateral displacements become functions of only the spatial coordinates x and y. Because the transverse displacement is in a dynamic state, the transverse displacement is a function of time t as well as the coordinates x and y. Therefore, the displacements can be represented by

$$u = u(x, y), \quad v = v(x, y), \quad w = w(t, x, y). \tag{1}$$

The position vector of point P after deformation of the membrane may be represented by

$$\mathbf{r} = (x + u)\mathbf{i} + (y + v)\mathbf{j} + w\mathbf{k}, \tag{2}$$

where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors in the x, y and z directions, respectively. The material derivative of the position vector $\mathbf{r}$ with respect to time leads to the velocity vector of point P given by

$$\mathbf{v} = V\left(1 + \frac{\partial u}{\partial x}\right)\mathbf{i} + V\frac{\partial v}{\partial x}\mathbf{j} + \left(\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x}\right)\mathbf{k}. \tag{3}$$

To consider geometric nonlinearity caused by the large deformation of the membrane, the strains should be represented as nonlinear functions of the displacements. According to the von Karman strain theory [14], the strain–displacement relations are given by

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \quad \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right). \tag{4}$$

On the other hand, the linearized stresses are used in this study. The linearized stresses are given by

$$\sigma_x = \frac{E}{1-v^2}\left(\frac{\partial u}{\partial x} + v\,\frac{\partial v}{\partial y}\right), \quad \sigma_y = \frac{E}{1-v^2}\left(\frac{\partial v}{\partial y} + v\,\frac{\partial u}{\partial x}\right), \quad \sigma_{xy} = \frac{E}{2(1+v)}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{5}$$

where E is Young's modulus and v is Poisson's ratio. Although the linearized stresses are used for modelling instead of the nonlinear stresses, the dynamic behaviour of the membrane can be well described because the geometric nonlinearity is due to large deformation.

The equations of motion can be derived from the extended Hamilton principle after the kinetic energy and the potential energy are obtained by using Eqs. (3)–(5). The derived equations of motion for the free vibration of the axially moving membrane may be expressed as

$$\rho V^2\,\frac{\partial^2 u}{\partial x^2} - \frac{\partial \sigma_x}{\partial x} - \frac{\partial \sigma_{xy}}{\partial y} = 0, \tag{6}$$

$$\rho V^2\,\frac{\partial^2 v}{\partial x^2} - \frac{\partial \sigma_y}{\partial y} - \frac{\partial \sigma_{xy}}{\partial x} = 0, \tag{7}$$

$$\rho\left(\frac{\partial^2 w}{\partial t^2} + 2V\,\frac{\partial^2 w}{\partial t \partial x} + V^2\,\frac{\partial^2 w}{\partial x^2}\right) - \frac{\partial}{\partial x}\left(\sigma_x\,\frac{\partial w}{\partial x} + \sigma_{xy}\,\frac{\partial w}{\partial y}\right) - \frac{\partial}{\partial y}\left(\sigma_y\,\frac{\partial w}{\partial y} + \sigma_{xy}\,\frac{\partial w}{\partial x}\right) = 0. \tag{8}$$

Since the stresses σ_x, σ_y and σ_{xy} are functions of only the in-plane displacements u and v, Eqs. (6) and (7) are dependent on u and v but independent of time t. This means that Eqs. (6) and (7) are equations for the steady-state equilibrium of the in-plane displacements. However, the equation of out-of-plane motion, given by Eq (8), is an equation for a dynamic state. It is interesting that Eqs. (6) and (7) are linear equations and these equations are coupled through only the in-plane displacements u and v. For this reason, Eqs. (6) and (7) can be solved independently without considering the equation of out-of-plane motion. On the other hand, Eq. (8) is a nonlinear equation where the out-of-plane displacement w is coupled to the in-plane displacements. Appearance of the in-plane stresses in the equation of out-of-plane motion implies that the in-plane stresses have an influence on the out-of-plane vibration of the membrane.

Two cases of the boundary conditions, which were introduced in Ref. [9], are also considered in this paper. The differences between these two cases are only the boundary conditions of the lateral direction, i.e., the y direction at $x = 0$ and L. All the boundary conditions except these ones are the same for both cases. In Case I, there is no friction between the membrane and the rollers in the lateral direction, while in Case II there is no slipping between them in the same direction. In other words, at the rollers, the membrane of Case I can freely move in the y direction but the membrane of Case II is fixed in this direction. The boundary conditions of Case I are written by

$$\sigma_x = T, \quad \sigma_{xy} = w = 0 \quad \text{at } x = 0, L, \tag{9}$$

$$\sigma_y = \sigma_{xy} = \sigma_y\frac{\partial w}{\partial y} + \sigma_{xy}\frac{\partial w}{\partial x} = 0 \quad \text{at } y = 0, b, \tag{10}$$

while the boundary conditions of Case II are written by

$$\sigma_x = T, \quad v = w = 0 \quad \text{at } x = 0, L, \tag{11}$$

$$\sigma_y = \sigma_{xy} = \sigma_y \frac{\partial w}{\partial y} + \sigma_{xy} \frac{\partial w}{\partial x} = 0 \quad \text{at} \quad y = 0, b. \tag{12}$$

3. Approximate solutions

In order to obtain approximate solutions from the equations of motion and the boundary conditions, the Galerkin method is used in this study. The Galerkin method requires the comparison functions as the basis functions, which should satisfy all the boundary conditions including the essential and natural boundary conditions. However, it is very hard to find the comparison functions to satisfy the boundary conditions given in Eqs. (9)–(12). To circumvent this difficulty, the equations of motion and the associated boundary conditions need to be transformed into the weak form because the weak form permits the admissible functions as the basis functions. Since the admissible functions satisfy only the essential boundary conditions, it is relatively easy to select the admissible functions as the basis functions compared to the comparison functions.

Before deriving the weak form, the weighting functions need to be defined. In this study, the weighting functions are defined as arbitrary functions to satisfy the essential boundary conditions. The weighting functions for u, v and w are denoted by $\bar{u}$, $\bar{v}$ and $\bar{w}$, respectively. As discussed before, since the equations of in-plane motion, Eqs. (6) and (7), have coupling between only the in-plane displacements u and v, they can be solved regardless of the equation of out-of-plane motion, Eq. (8). After the in-plane displacements are determined from Eqs. (6) and (7), the stresses σ_x, σ_y and σ_{xy} can be regarded as prescribed functions of x and y. Then, Eq. (8) becomes a linear partial differential equation with an unknown displacement w. For this reason, two weak forms are derived in this paper: one is for the in-plane motion and the other is for the out-of-plane.

The weak form for the in-plane motion may be obtained by multiplying Eqs. (6) and (7) by $\bar{u}$ and $\bar{v}$, respectively, summing them, integrating the resultant equation over the area A and applying integration by parts. The weak form for the in-plane motion may be represented by

$$\begin{aligned} &\int_A \left[\rho V^2 \left(\bar{u} \frac{\partial^2 u}{\partial x^2} + \bar{v} \frac{\partial^2 v}{\partial x^2} \right) + \sigma_x \frac{\partial \bar{u}}{\partial x} + \sigma_y \frac{\partial \bar{v}}{\partial y} + \sigma_{xy} \left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right) \right] \mathrm{d}A \\ &\quad = T \int_0^b \left(\bar{u}|_{x=L} - \bar{u}|_{x=0} \right) \mathrm{d}y. \end{aligned} \tag{13}$$

In a similar way, the weak form for the out-of-plane motion can be derived. The weak form for the out-of-plane motion can be written as

$$\begin{aligned} &\int_A \left[\rho \bar{w} \left(\frac{\partial^2 w}{\partial t^2} + 2V \frac{\partial^2 w}{\partial t \, \partial x} + V^2 \frac{\partial^2 w}{\partial x^2} \right) \right. \\ &\quad \left. + \sigma_x \frac{\partial \bar{w}}{\partial x} \frac{\partial w}{\partial x} + \sigma_y \frac{\partial \bar{w}}{\partial y} \frac{\partial w}{\partial y} + \sigma_{xy} \left(\frac{\partial \bar{w}}{\partial x} \frac{\partial w}{\partial y} + \frac{\partial \bar{w}}{\partial y} \frac{\partial w}{\partial x} \right) \right] \mathrm{d}A = 0, \end{aligned} \tag{14}$$

During derivation of the weak forms, the natural boundary conditions, which are related to the stresses, have already been considered. Therefore, the admissible functions can be used as the basis functions for the in-plane and out-of-plane displacements. It should be noted that the derived weak forms could be applicable for both Cases I and II. Only the difference between Cases I and II is the choice of the basis functions for the lateral displacement v.

The in-plane and out-of-plane displacements may be approximated in finite-dimensional function spaces, in which the basis functions consist of the admissible functions. Eqs. (9)–(12) show that u has no essential boundary condition for both Cases I and II and v has the essential boundary conditions only for Case II. Hence, the approximate solutions for the in-plane displacements can be written in terms of the Legendre polynomials or trigonometric functions:

$$u = \sum_{i=0}^{N_x} \sum_{j=0}^{N_y} T_{ij}^{u} X_i^{u}(x) Y_j(y), \quad v = \sum_{i=0}^{N_x} \sum_{j=0}^{N_y} T_{ij}^{v} X_i^{v}(x) Y_j(y), \tag{15}$$

where N_x and N_y are the total numbers of the basis functions for the in-plane displacements in the x and y directions, respectively; T_{ij}^u and T_{ij}^v are unknown coefficients to be determined; and $X_i^u(x)$, $X_i^v(x)$ and $Y_j(y)$ may be given by

$$X_i^u(x) = \sum_{r=0}^{R_1} (-1)^r \frac{(2i-2r)!}{2^i r!(i-r)!(i-2r)!} \left(\frac{2x}{L} - 1\right)^{i-2r}, \quad i = 0, 1, \ldots, N, \tag{16}$$

$$X_i^v(x) = \begin{cases} X_i^u(x) & \text{for Case I}, \\ \sin \frac{(i+1)\pi x}{L} & \text{for Case II}, \end{cases} \quad i = 0, 1, \ldots, N, \tag{17}$$

$$Y_j(y) = \sum_{r=0}^{R_2} (-1)^r \frac{(2j-2r)!}{2^j r!(j-r)!(j-2r)!} \left(\frac{2y}{b} - 1\right)^{j-2r}, \quad j = 0, 1, \ldots, N, \tag{18}$$

in which

$$R_1 = \begin{cases} i/2 & \text{if } i \text{ is even}, \\ (i-1)/2 & \text{if } i \text{ is odd}, \end{cases} \quad R_2 = \begin{cases} j/2 & \text{if } j \text{ is even}, \\ (j-1)/2 & \text{if } j \text{ is odd}, \end{cases} \tag{19}$$

Note that w has only the essential boundary conditions at $x = 0$ and L while it has only the natural boundary conditions at $y = 0$ and b. Therefore, the approximate solution of w may be expressed as

$$w = \sum_{p=0}^{M_x} \sum_{q=0}^{M_y} T_{pq}^{w}(t) Y_q(y) \sin \frac{(p+1)\pi x}{L}, \tag{20}$$

where M_x and M_y are the total number of the basis functions for the out-of-plane displacement in the x and y directions. The weighting functions corresponding to u, v and w can be expressed as

$$\bar{u} = \sum_{m=0}^{N_x}\sum_{n=0}^{N_y} \bar{T}^u_{mn} X^u_m(x) Y_n(y), \quad \bar{v} = \sum_{m=0}^{N_x}\sum_{n=0}^{N_y} \bar{T}^v_{mn} X^v_m(x) Y_n(y),$$

$$\bar{w} = \sum_{r=0}^{M_x}\sum_{s=0}^{M_y} \bar{T}^w_{rs}(t) Y_s(y) \sin\frac{(r+1)\pi x}{L}, \tag{21}$$

where $\bar{T}^u_{mn}$ and $\bar{T}^v_{mn}$ are arbitrary constants and $\bar{T}^w_{rs}(t)$ is an arbitrary function of time.

The discretized equations of motion can be obtained by substituting Eqs. (15), (20) and (21) into Eqs. (13) and (14). Collecting all the terms of the resultant equations with respect to $\bar{T}^u_{mn}$, $\bar{T}^v_{mn}$ and $\bar{T}^w_{rs}(t)$, the coefficients of $\bar{T}^u_{mn}$, $\bar{T}^v_{mn}$ and $\bar{T}^w_{rs}(t)$ yield discretized equations, which can be written by two matrix-vector equations. One is the equation of in-plane force equilibrium in a steady state and the other is the equation of out-of-plane motion in a dynamic state. The discretized equations of in-plane force equilibrium may be rewritten as

$$\mathbf{K}^{uv}\mathbf{T}^{uv} = \mathbf{F}^{uv}, \tag{22}$$

where $\mathbf{K}^{uv}$ is the stiffness matrix for the in-plane motion, $\mathbf{F}^{uv}$ is the load vector, and $\mathbf{T}^{uv}$ is given by

$$\mathbf{T}^{uv} = \begin{Bmatrix} \mathbf{T}^u \\ \mathbf{T}^v \end{Bmatrix} \tag{23}$$

in which

$$\mathbf{T}^u = \{T^u_{00}, T^u_{10}, \ldots, T^u_{N_x0}, T^u_{01}, T^u_{11}, \ldots, T^u_{N_x1}, \ldots, T^u_{0N_y}, T^u_{1N_y}, \ldots, T^u_{N_xN_y}\}^{\mathrm{T}},$$

$$\mathbf{T}^v = \{T^v_{00}, T^v_{10}, \ldots, T^v_{N_x0}, T^v_{01}, T^v_{11}, \ldots, T^v_{N_x1}, \ldots, T^v_{0N_y}, T^v_{1N_y}, \ldots, T^v_{N_xN_y}\}^{\mathrm{T}}. \tag{24}$$

It should be pointed out that Eq. (22) cannot be solved without a special treatment because the $\mathbf{K}^{uv}$ matrix has no inverse matrix. This is caused by the fact that the membrane can have a rigid-body motion in the xy plane. To get rid of the rigid-body motion, some rows and columns, which have only zero elements or are dependent on the other rows or columns, should be deleted from the $\mathbf{K}^{uv}$ matrix. After the coefficients T^u_{ij} and T^v_{ij} are computed from Eq. (22), the in-plane stresses σ_x, σ_y and σ_{xy} as well as the in-plane displacements u and v become functions of x and y. On the other hand, the discretized equations of out-of-plane motion may be written as

$$\mathbf{M}^w\ddot{\mathbf{T}}^w + 2V\mathbf{G}^w\dot{\mathbf{T}}^w + (V^2\mathbf{H}^w + \mathbf{K}^w)\mathbf{T}^w = \mathbf{0}, \tag{25}$$

where $\mathbf{M}^w$ is the mass matrix for the out-of-plane motion, $\mathbf{G}^w$ is the matrix related to the gyroscopic force, $\mathbf{H}^w$ is the matrix related to the centrifugal force, $\mathbf{K}^w$ is the structural stiffness matrix, and $\mathbf{T}^w$ is defined as

$$\mathbf{T}^w = \{T^w_{00}, T^w_{10}, \ldots, T^w_{M_x0}, T^w_{01}, T^w_{11}, \ldots, T^w_{M_x1}, \ldots, T^w_{0M_y}, T^w_{1M_y}, \ldots, T^w_{M_xM_y}\}^{\mathrm{T}}. \tag{26}$$

To obtain the natural frequencies and mode shapes for the out-of-plane vibration of the axially moving membrane, the eigenvalue problem should be derived from Eq. (25). The solution of Eq. (25) can be assumed as

$$\mathbf{T}^w = \mathbf{T}_0 \mathrm{e}^{\lambda_n t}, \tag{27}$$

where $\mathbf{T}_0$ is the eigenvector and λ_n is the eigenvalue or the complex natural frequency. Note that the natural frequency in a general sense, ω_n, is the imaginary part of the eigenvalue λ_n. Substitution of Eq. (27) into Eq (25) leads to the eigenvalue problem given by

$$(\lambda_n^2\mathbf{M}^w + 2V\lambda_n\mathbf{G}^w + V^2\mathbf{H}^w + \mathbf{K}^w)\mathbf{T}_0 = \mathbf{0}. \tag{28}$$

The complex natural frequencies of the membrane can be computed from

$$\det(\lambda_n^2\mathbf{M}^w + 2V\lambda_n\mathbf{G}^w + V^2\mathbf{H}^w + \mathbf{K}^w) = 0. \tag{29}$$

For a given eigenvalue λ_n, Eq. (28) yields the corresponding eigenvector $\mathbf{T}_0$. This eigenvector is used to determine the associated mode shape from Eq. (20).

4. Analysis and discussion

Before discussing the natural frequencies and mode shapes of the membrane, it is convenient for simplicity of discussion to define the dimensionless eigenvalue $\tilde{\lambda}$ and the dimensionless velocity $\tilde{V}$ as follows:

$$\tilde{\lambda} = \lambda_n \frac{L}{\pi}\sqrt{\frac{\rho}{T}}, \quad \tilde{V} = V\sqrt{\frac{\rho}{T}}. \tag{30}$$

The dimensionless natural frequency, denoted by $\tilde{\omega}$, can be defined by the real part of the dimensionless eigenvalue $\tilde{\lambda}$. At this point, it is necessary to define a mode (k, l) of the given membrane. In the (k, l) mode, k and l represent the orders of mode complexities in the x and y directions, respectively. In fact, k is the number of nodal lines (excluding the end lines at the rollers) parallel to the rollers or the y-axis.

4.1. Effects of the translating speed on the natural frequencies

First, the effects of the translating speed on the natural frequencies are investigated. Fig. 2 shows the dimensionless eigenvalues $\tilde{\lambda}$ of a membrane with $L = b$ for the variation of the dimensionless translating speed $\tilde{V}$ when the membrane has the boundary conditions of Case I. Fig. 2a is for the imaginary parts of $\tilde{\lambda}$, i.e., $\tilde{\omega}$ while Fig. 2b is for the real parts of $\tilde{\lambda}$. If the modes for the membrane of Case I have the same number of nodal lines parallel to the rollers, their natural frequencies are identical regardless of the mode complexity in the y direction. This fact can be identified in Fig. 2a, where the lines of the (k, l) modes for a given value of k and $l = 1, 2, \ldots$ overlap. It is also found that the dimensionless natural frequency of the (k, l) mode has a value of $k + 1$ when $\tilde{V} = 0$, decreases with $\tilde{V}$, and becomes zero when $\tilde{V} = 1$. These features for the membrane of Case I are the same as those for the axially moving string discussed in Ref. [6]. It is well known that the speed when the natural frequency becomes zero is called the critical speed. Thus, the critical speed for the membrane of Case I is $\tilde{V} = 1$. As shown in Fig. 2b, the imaginary parts of $\tilde{\lambda}$ is zero until $\tilde{V}$ increases to the critical speed. Therefore, the translating membrane of Case I is dynamically stable when the translating speed is below the critical speed.

Next, the effects of the translating speed on the natural frequencies are considered for the membrane of Case II. For the membrane of Case II, the dimensionless eigenvalues for the

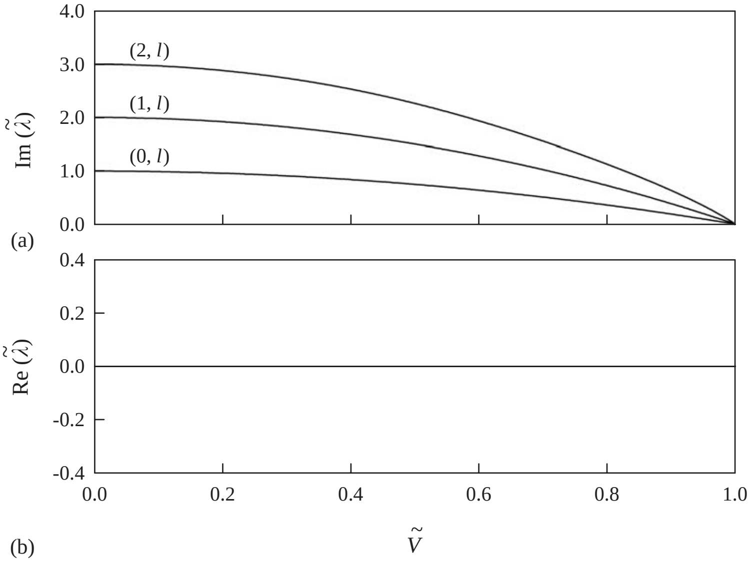

Fig. 2. Dimensionless eigenvalues $\tilde{\lambda}$ for the variation of the dimensionless translating speed $\tilde{V}$ when a membrane with $L = b$ has boundary conditions of Case I: (a) the imaginary parts; (b) the real parts.

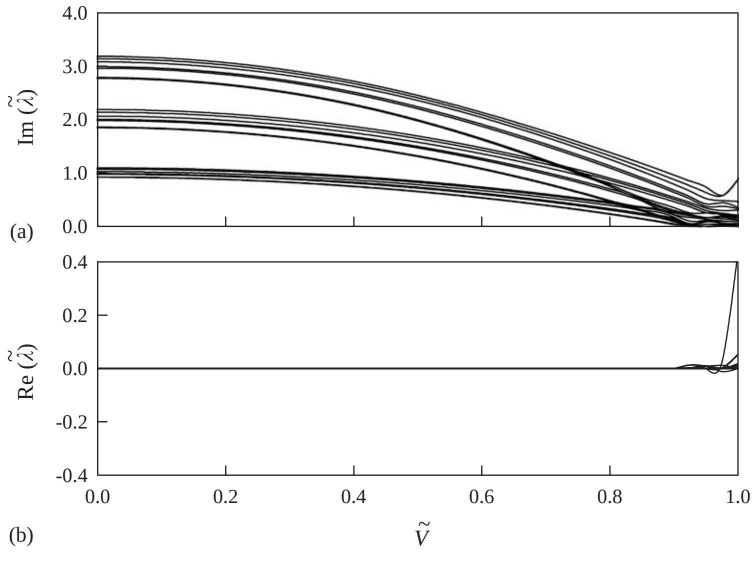

Fig. 3. Dimensionless eigenvalues $\tilde{\lambda}$ for the variation of the dimensionless translating speed $\tilde{V}$ when a membrane with $L = b$ has boundary conditions of Case II: (a) the imaginary parts; (b) the real parts.

variation of the dimensionless translating speed are shown in Fig. 3, in which Fig. 3a is for the imaginary parts of the eigenvalues and Fig. 3b is for the real parts. Similar to the membrane of Case I, the membrane of Case II has the decreasing natural frequencies with the translating speed except in the neighbourhood of $\tilde{V} = 1$. However, the different behaviours of the natural frequencies are observed between the membranes of Cases I and II if Figs. 2 and 3 are compared. For a given value of k, the natural frequencies of the (k, l) modes for Case II have distinct values from each other if l is different. Therefore, the natural frequencies of the (k, l) mode for a given value of k and $l = 1, 2, \ldots$ do not overlap in Fig. 3a. It should be also noted from Fig. 3b that the real parts of the eigenvalues become positive even when $\tilde{V} < 1$. This means that the membrane of Case II has dynamic instability at a lower translating speed that the membrane of Case I. Consequently, the dynamic characteristics for the membrane of Case II may show quite different behaviours from those for a string.

4.2. Mode shapes of the moving membrane

The mode shapes are examined for the axially moving membrane of Case I. Since there is no friction between the membrane and the rollers in the y direction or in the direction parallel to the rollers, it can be inferred that the membrane does not have tension in this direction. Fig. 4 illustrates the mode shapes of the axially moving membrane of Case I when the dimensionless translating speed is $\tilde{V} = 0.5$. The modes (0, 1), (0, 2) and (0, 3), shown in Figs. 4a–c, have no nodal line parallel to the y-axis, while the (1, 1), (1, 2) and (1, 3), shown in Figs. 4d–f, have one nodal line.

The mode shapes of the membrane of Case II are also examined. Fig. 5 illustrates the mode shapes of the membrane of Case II when $\tilde{V} = 0.5$. In the membrane of Case II, there is no slipping between the membrane and the rollers in the y direction. Since the membrane does not slip at the rollers, friction forces exist between the membrane and the rollers. The existence of the friction forces implies that the membrane is subjected to tension in the y direction.

Comparing the mode shapes between Cases I and II, it is seen that the mode shapes of Case II have less fluctuation in the y direction than those of Case I. This is caused by the fact that the membrane of Case II has tension in the y direction but the membrane of Case I has no tension in this direction.

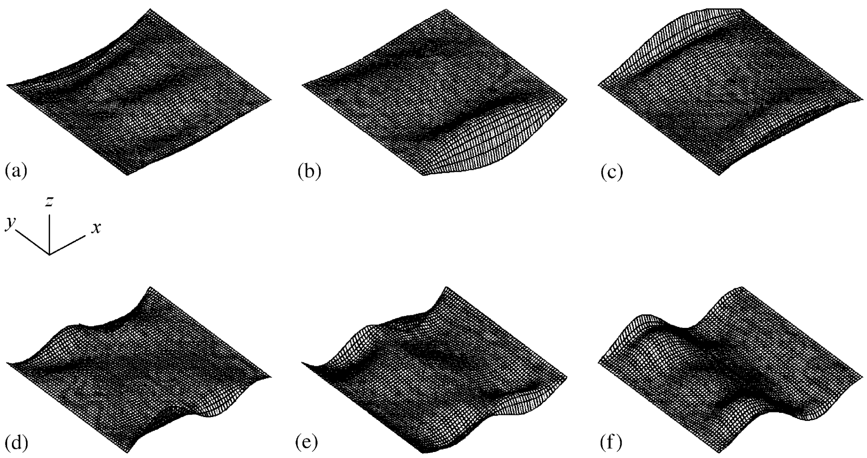

Fig. 4. Mode shapes for the moving membrane of Case I when $\tilde{V} = 0.5$ and $L = b$: (a) the (0, 1) mode; (b) the (0, 2) mode; (c) the (0, 3) mode; (d) the (1, 1) mode; (e) the (1, 2) mode and (f) the (1, 3) mode.

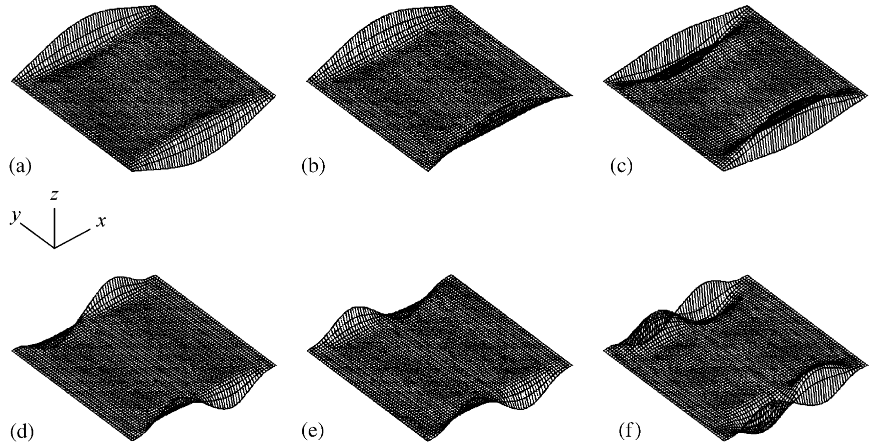

Fig. 5. Mode shapes for the moving membrane of Case II when $\tilde{V} = 0.5$ and $L = b$: (a) the (0, 1) mode; (b) the (0, 2) mode; (c) the (0, 3) mode; (d) the (1, 1) mode; (e) the (1, 2) mode and (f) the (1, 3) mode.

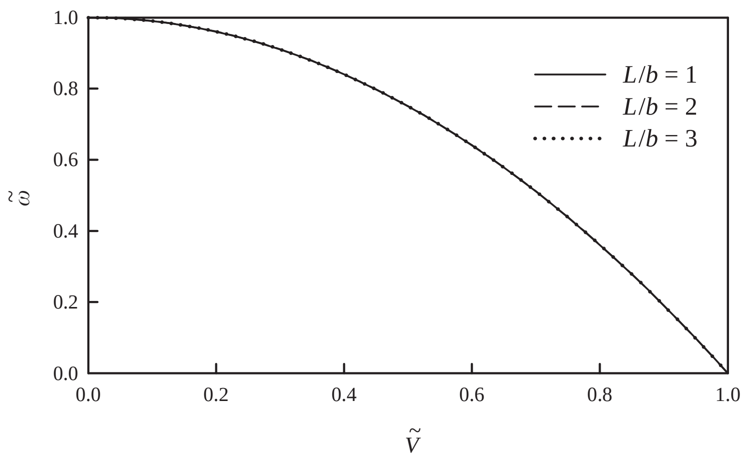

Fig. 6. Comparison of the lowest natural frequencies for the membrane of Case I when the aspect ratios are $L/b = 1$ (——), 2 (– – – –) and 3 (··· ··· ···).

4.3. Effects of the aspect ratio on the natural frequencies

The effects of the aspect ratio on the natural frequencies of the membrane are examined for both Cases I and II. In order to observe the effects of the aspect ratio, the natural frequencies are computed decreasing the width b with the fixed length L. Figs. 6 and 7 show the lowest natural frequencies versus the translating speed for the membranes of Cases I and II, respectively, when the aspect ratios L/b are 1, 2 and 3. Here, the lowest natural frequency represents the natural frequency corresponding to the (0, 1) mode. As illustrated in Fig. 6, the lines of the natural frequencies for $L/b = 1$, 2 and 3 overlap regardless of the translating speed if the membrane has

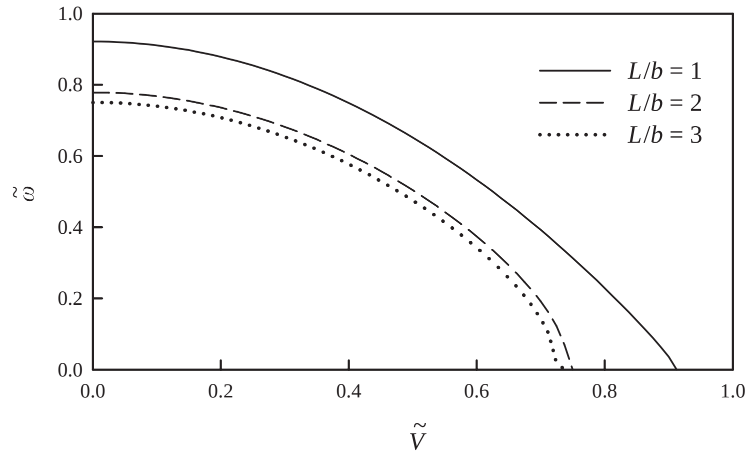

Fig. 7. Comparison of the lowest natural frequencies for the membrane of Case II when the aspect ratios are $L/b = 1$ (——), 2 (– – – –) and 3 (⋯ ⋯ ⋯).

the boundaries of Case I. This means that the aspect ratio of the membrane has no influence on the lowest natural frequency and the critical speed for the membrane of Case I. In contrast to the membrane of Case I, the membrane of Case II has the lowest natural frequencies and critical speed affected by the aspect ratio, as shown in Fig. 7. It is found from the figure that the critical speed as well as the lowest natural frequency diminishes with the aspect ratio. This tendency is the same as that of the in-plane vibration presented in Ref. [9].

5. Conclusions

The dynamic characteristics of the out-of-plane vibration for the axially moving membrane were analyzed in this paper. The equations of in-plane and out-of-plane motions were derived by considering two sets of boundary conditions: Cases I and II. After the equations of motion and the associated boundary conditions were transformed to the weak forms, the Galerkin method was applied to obtain approximate solutions.

From the analysis of the natural frequencies and mode shapes for the axially moving membrane, the following conclusions are obtained:

(1) The natural frequencies and the critical speed for the membrane of Case II are lower than those for the membrane of Case I. In other words, the membrane of Case II may become unstable even when the dimensionless velocity is less than 1, i.e., $\tilde{V} < 1$; however, the membrane of Case I is stable when $\tilde{V} < 1$.
(2) For a given mode complexity in the direction perpendicular to the rollers, the natural frequencies for the membrane of Case I are the same irrespective of the mode complexity in the direction parallel to the rollers; however, those for the membrane of Case II are distinct for different mode complexities in this direction.

(3) The mode shapes for the membrane of Case II have less fluctuation in the direction parallel to the rollers than those of Case I, because the membrane of Case II has tension in this direction while the membrane of Case I has no apparent tension.
(4) The aspect ratio has no influence on the natural frequencies and the critical speed for the membrane of Case I while it has an influence on those for the membrane of Case II.

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